



## Design of Basic Logic Gates by Triple Quantum Rings

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### ABSTRACT

In this paper, we study the electron transport in the triple quantum rings in the presence of a magnetic flux threaded through each ring. The triple quantum rings is connected to the semi-infinite external electrodes. The Hamiltonian of the system is written by using the tight-binding model and the electron transmission probability is calculated by the Green's function method. The electron current passing the triple quantum rings is calculated in terms of the applied bias voltage. We propose different logic gates such as AND, OR, NOT, XOR, NOR, XNOR and NAND by applying the gate voltages to different atomic sites of the triple rings.

## 1. Introduction

In the recent years, the electron transport through quantum confined systems has investigated by some groups due to its use in the study of nanoelectronics and spintronics [1-16]. On the other hand, with great progress in nanofabrication techniques, the quantum confined systems such as quantum rings can be used in designing nanodevices which are treated as the fundamental building blocks for future generation of nanoelectronics. The key idea of designing nanodevices based on the quantum rings is the concept of quantum interference effect [17]. A normal metal mesoscopic ring is a very nice example where the electronic motion is confined and the transport becomes predominantly coherent. Probably, the first attention to the quantum transport back to the Aviram and Rantner's study in 1974 [1]. They have investigated a rectifier based on the single organic molecule. The response of such a molecule to an applied field has been calculated, and rectifier properties have been appeared.

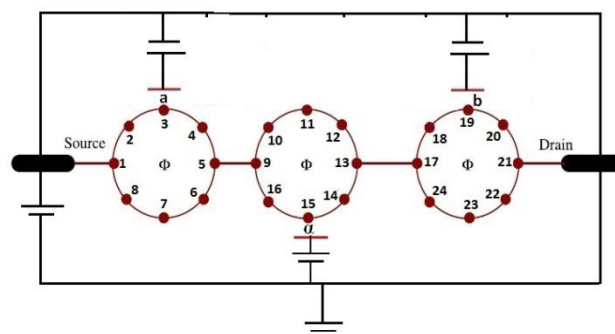
Up to now, the electron transport in the quantum rings have been studied theoretically and experimentally by a number of research groups [2-16]. The electronic transport of a noninteracting quantum ring side coupled to a quantum wire is studied in the presence of the external magnetic fields via a single-band tunneling tight-binding Hamiltonian [4].

Maiti have designed some logic gates by inserting a single or double quantum rings including a few atoms between two external electrodes such as source and drain [6-14]. A thin magnetic flux passes through the ring changes the phase of electron wave function propagating the ring circumference and causes the constructive or destructive interference occurs which influences the transmission probability. The conductance-spectrum and the current-voltage characteristic were obtained for different ring-electrode couplings. For the design of each logic gate, the gate voltages have been applied to the two or more atomic sites as the inputs of logic gates. The tight-binding model has been used to describe the system and the current-voltage characteristics have been calculated by using the Green's function formalism.

In this paper, we investigate the electron transport in the triple quantum rings which are connected in series. The electron current passing through the system is calculated as a function of applied voltage by using the tight-binding model and the Green's function formalism. In ballistic approximation, different logic gates are designed by applying different gate voltages to different locations of the system.

## 2. Theoretical Model

Figure 1 shows the triple quantum rings which are connected in series between two semi-infinite one-dimensional (1D) metallic electrodes or the same source and drain. A magnetic flux is threaded through each ring. The gate voltages are applied to the special atomic sites which their locations depend on the type of the gate that we want to design. We assume that all the gate voltages operate on the nearest atomic site of the electrode plate. This configuration is operated as an AND gate which the potentials applied to the atomic sites a and b are the two inputs of its.



**Fig. 1** (Color online). Schematic view of a triple quantum rings connected to the external electrodes and operated as an AND gate. The voltages  $V_a$ ,  $V_b$  and  $V_\alpha$  are applied to the atomic sites a, b and  $\alpha$ .

The transmission probability of an electron across the triple rings (i.e.,  $T$ ) can be calculated in terms the Green's function of the triple rings and its coupling with source and drain electrodes as follow [3, 5]

$$T = \text{Tr} [I_S^r G_R^r I_D G_R^a], \quad (1)$$

where  $G_R^r$  and  $G_R^a$  are respectively the retarded and advanced Green's functions of the triple quantum rings including the source and drain effects. The parameters  $I_S^r$  and  $I_D^r$  describe the coupling of the triple rings to the source and drain, respectively. In generally, the Green's function of system including the triple quantum rings, source and drain, is expressed as,

$$G = (E - H)^{-1}, \quad (2)$$

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where E is the injecting energy of the source electron and H is also the Hamiltonian of the full system. Since the full system is composed of two semi-infinite electrodes, for the calculation of the Green's function it is necessary to invert an infinite matrix. Then, the full system is divided into sub-matrices corresponding to the individual sub-systems. The effective Green's function for the triple quantum rings can be written as,

$$G_R = (E - H_R - \Sigma_S - \Sigma_D)^{-1}, \tag{3}$$

where  $H_R$  is the Hamiltonian of the triple quantum rings. The parameters  $\Sigma_S$  and  $\Sigma_D$  in the above equation represent the self-energies due to the coupling of the triple rings to the source and drain, respectively [3]. Indeed, all the information of the coupling are included into these self-energies. In the case of the non-interacting electrons, the Hamiltonian can be written as

$$H_R = \sum_i (\epsilon_i + V_a \delta_{i\alpha} + V_b \delta_{i\beta}) c_i^\dagger c_i + \sum_{ij} t (c_i^\dagger c_j e^{i\theta} + c_j^\dagger c_i e^{-i\theta}) \tag{4}$$

where  $\epsilon_i$  is the energy of the  $i$ th site,  $V_i$ ,  $i = a, b, \alpha$  are the gate voltages which are applied to these sites. The  $c_i^\dagger$  ( $c_i$ ) is the creation (annihilation) operator of an electron at the site  $i$ th and  $t$  is the hopping integral between the two neighboring sites in each ring. For simplicity, the hopping strength between the two connector sites of the each two rings is also set to  $t$ .  $\theta$  in the Hamiltonian [Eq. (4)] is the phase factor which is related to the magnetic flux (i.e.,  $\phi$ ) as  $\theta = 2\pi\phi/N\phi_0$  where  $\phi_0 = h/e$  is the quantum flux and  $N$  is the number of the atomic sites of each ring. In addition,  $h$  and  $e$  are the Planck constant and the charge of the electron, respectively.

The Hamiltonian of the one-dimensional electrodes is also similar to Eq. (4), except that the phase factor (i.e.,  $\theta$ ) is zero. In this case, the on-site energy is indicated by  $\epsilon_i'$  and  $t'$  is the nearest-neighbor hopping integral of the one-dimensional electrodes. The current passing the triple quantum rings is calculated as a function of the applied voltage by using the relation

$$I = \frac{e}{\pi h} \int_{E_F - eV/2}^{E_F + eV/2} T(E) dE, \tag{5}$$

where  $E_F$  is the equilibrium Fermi energy.

### 3. Results and Discussion

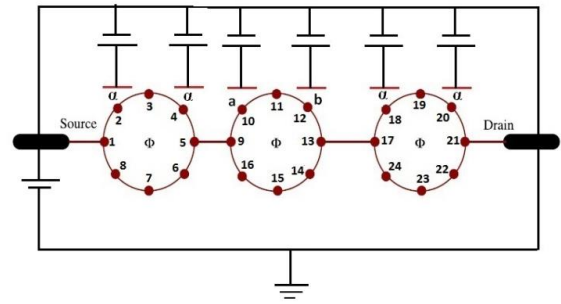
In this section, we represent the numerical study of the electron transport in the triple quantum rings. The electron current is calculated by using the Green's function method. For numerical calculation, the on-site energy of the triple rings (i.e.,  $\epsilon$ ) and the side attached electrodes (i.e.,  $\epsilon'$ ) are chosen as  $\epsilon = \epsilon' = 0$ . The nearest-neighbor hopping strength of them are supposed to be  $t=3$  and  $t' = 4$ . In addition, for the simplicity, the Fermi energy is supposed to be  $E_F = 0$ , the constant parameters  $e$  and  $h$  are taken as 1. For the constructive interference of the electron wave functions, the magnetic flux is set to the  $\phi = \phi_0/2$  which is equal to the 0.5 due to the chosen values for the constant parameters. We suppose that the strengths of the coupling of the triple quantum rings and the external electrodes are strong (i.e.,  $\tau_{S(D)} \sim t$ ) and they are chosen  $\tau_S = \tau_D = 2.5$ .

In Fig. 1, for the design of the AND gate, the gate voltages  $V_a$ ,  $V_b$  and  $V_\alpha$  are applied to the specific atomic sites. In this case, the potential  $V_\alpha$  is fixed to 2 and the gate voltages  $V_a$  and  $V_b$  are considered as the two input voltages. The current passing through the triple rings is calculated in the particular bias voltage (i.e.,  $V=6$ ) and listed in the Table 1. It is observed that the current is non zero when the two input voltages have the non-zero values and then the triple rings behave as the AND gate. The triple quantum rings again behave as the AND gate when we change the location of the applied voltages  $V_a$ ,  $V_b$  and  $V_\alpha$  from that is shown in Fig. 1 to the atomic sites  $a=3$ ,  $b=19$  and  $\alpha=11$ .

**Table 1** The truth table of the proposed AND gate. The current I is computed at the bias voltage 6

Input-I ( $V_a$ )	Input-I ( $V_b$ )	Current (I)
0	0	0
2	0	0
0	2	0
2	2	0.9611

To design of an OR gate, the applied voltages in Fig. 1 are changed according to its shown in Fig. 2. Table 2 lists the calculated current at the bias voltage  $V=6$  for the different input gate voltages. It is clear that the triple rings behave as the OR gate. The other configurations of the OR gate can be obtained by applying the input voltages to atomic sites  $a=2$  and  $b=4$  and the constant voltage to atomic sites  $\alpha=11$  and 19 and also for  $a=2$ ,  $b=4$  and  $\alpha=11$  and 23.

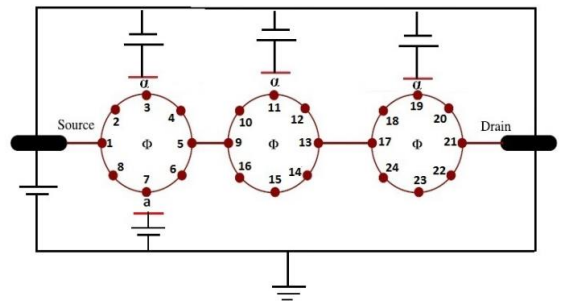


**Fig. 2** (Color online). Schematic view of a triple quantum rings operated as an OR gate

**Table 2** The truth table of the proposed OR gate. The current I is computed at the bias voltage 6

Input-I ( $V_a$ )	Input-I ( $V_b$ )	Current (I)
0	0	0
2	0	1.4682
0	2	1.4682
2	2	2.6514

Figure 3 shows a NOT gate which can be designed by the triple quantum rings. The gate voltage  $V_a$  is considered as the gate input while the constant potential voltage  $V_\alpha$  is applied to the three atomic sites  $\alpha=3, 11$  and 19. The current values of the triple rings for this configuration are listed in Table 3 for two possible values of the input voltages. It is deduced that the triple rings act as a NOT gate. Applying the input gate voltage (i.e.,  $V_a$ ) to the atomic site  $a=15$  also lead to the same result.

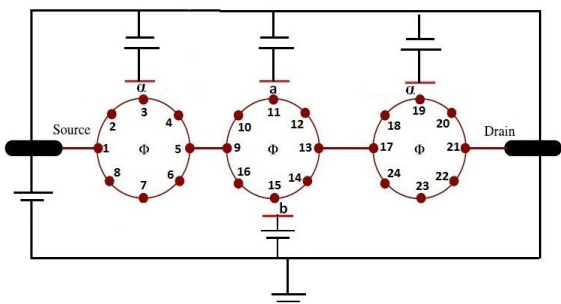


**Fig. 3** (Color online). Schematic view of a triple quantum rings operated as a NOT gate

**Table 3** The truth table of the proposed NOT gate. The current I is computed at the bias voltage 6

Input-I ( $V_a$ )	Current (I)
0	0.9611
2	0

The triple quantum rings can be operated as a XOR gate if the applied potentials to the atomic sites are performed as shown in Fig. 4. The current values through the triple rings is calculated for  $V=6$  and are listed in Table 4. It is seen that the output current is established when only one of the two input voltages  $V_a$  and  $V_b$  is zero. Therefore the triple ring operates as a XOR gate. The change of the location of the applied voltages  $V_a$  and  $V_b$  to the atomic sites  $a=19$ ,  $b=23$  and the constant voltage  $V_\alpha$  to  $\alpha=3$  and 11 yields other configuration of the XOR gate.

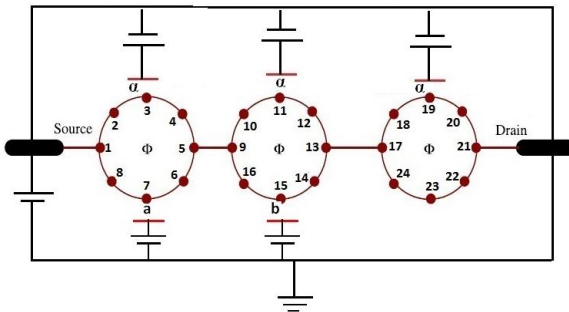


**Fig. 4** (Color online). Schematic view of a triple quantum rings operated as a XOR gate

**Table 4** The truth table of the proposed XOR gate. The current I is computed at the bias voltage 6

Input-I (Va)	Input-I (Vb)	Current (I)
0	0	0
2	0	0.9611
0	2	0.9611
2	2	0

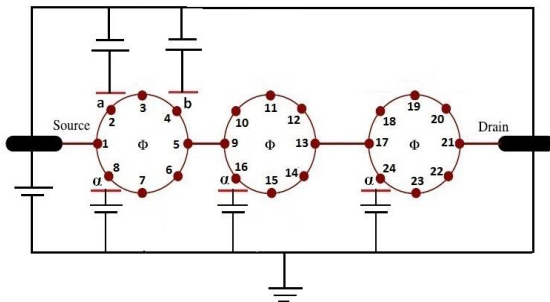
Now, we want to investigate the triple quantum rings as a NOR logic gate. A NOR gate is the result of the negation of the OR operator. It can also be seen as an AND gate with all the inputs inverted. A high output ( $\neq 0$ ) results if both the inputs to the gate are low (0). In Fig. 5, a NOR logic gate is proposed by the triple quantum rings which are connected to the external electrodes. Table 5 shows the truth table of the mentioned NOR gate. It is seen that the output current is zero when at least one of the two input voltages is non-zero. By changing the location of applied gate voltages as shown in Fig. 6, we have a XNOR logic gate which the output is high ( $\neq 0$ ) when both inputs Va and Vb are high (i.e.,  $\neq 0$ ) and when neither Va nor Vb is high. The output current values are calculated for different input voltages and are listed in Table 6.



**Fig. 5** (Color online). Schematic view of a triple quantum rings operated as a NOR gate

**Table 5** The truth table of the proposed NOR gate. The current I is computed at the bias voltage 6

Input-I (Va)	Input-I (Vb)	Current (I)
0	0	0.9611
2	0	0
0	2	0
2	2	0

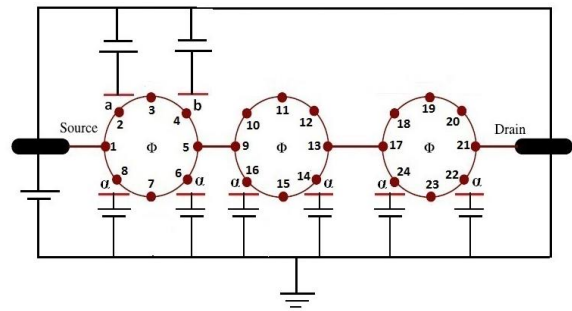


**Fig. 6** (Color online). Schematic view of a triple quantum rings operated as a XNOR gate

**Table 6** The truth table of the proposed XNOR gate. The current I is computed at the bias voltage 6

Input-I (Va)	Input-I (Vb)	Current (I)
0	0	0.6484
2	0	0
0	2	0
2	2	0.2812

Finally, as shown in Fig. 7, a NAND gate is designed by the triple rings. A NAND gate is a logic gate which produces a false output (0) only if all its inputs are true ( $\neq 0$ ). In other hand, a NAND gate is an inverted AND gate. Table 7 is the truth table of this configuration. It can be seen that the maximum value corresponds to the situation which both two input voltages are zero. In the above proposed NAND gate, due to the symmetry of the triple rings structure, it is clear that if the input voltages Va and Vb are applied to the atomic sites a=18 and b=20, we can see again a NAND gate behavior.



**Fig. 7** (Color online). Schematic view of a triple quantum rings operated as a NAND gate

**Table 7** The truth table of the proposed NAND gate. The current I is computed at the bias voltage 6

Input-I (Va)	Input-I (Vb)	Current (I)
0	0	2.6514
2	0	1.2604
0	2	1.1045
2	2	0

**4. Conclusion**

In this work, we have designed different classical logic gates based on the triple quantum rings which is connected to the semi-infinite one-dimensional external electrodes. A magnetic flux is threaded through the each ring. It is supposed that there are no any disorder and scattering in the triple rings and also the temperature is too low. Then, the quantum interference which is the main concept in designing of different gates cannot be disturbed in the ring circumference. The electron current is calculated by the Green’s function formalism for each configuration of the triple ring including input and fixed potentials. The all basic logic gates are designed by change the location of applied voltages in the triple rings.

**Acknowledgements**

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